Econ 201 Section 5 - Problem Set 1

Due 1/24 by Start of Class - Graded for ACCURACY

Problem 1

Answer each of the following questions as TRUE, FALSE, or UNCERTAIN, and justify your answer.

(a) If a production function $f(K, L)$ displays constant returns to scale, then $f_{KL} = \frac{\partial^2 f(K,L)}{\partial K \partial L}$ must be positive.

(b) If the firm is cost-minimizing and is at an optimum, the marginal cost is the same whether it changes only labor, only capital, or both.

(c) When marginal cost decreases, average cost also decreases. When marginal cost increases, average cost also increases.

(d) The minimum point of ATC curve is right above the minimum point of AVC curve (at the same y).

Problem 2

Consider a firm that has the production function $f(L, K) = L^{1/2} + K^{1/2}$.

(a) Does this production function satisfy monotonicity?

(b) Calculate the marginal products of of labor and capital. Does the production function satisfy diminishing marginal product for each input?

(c) What is the Technical Rate of Substitution for this firm? Explain the meaning of this in words.

(d) Does this firm have increasing, constant, or decreasing returns to scale?

Problem 3

Consider the following production functions:

(i) $f(K, L) = K^{1/2} L^{3/4}$

(ii) $f(K, L) = K + L^2$

(iii) $f(K, L) = max\{K, L\}$

Answer the following questions for all three production functions.

(a) Find the marginal products of labor and capital. Are the increasing, constant, or decreasing?

(b) Draw the isoquants. These need not be precise, but should capture the general shape the isoquants would have and have some justification as to why they would possess this shape.

(c) Is the production function convex?

(d) Does the production function display increasing, constant or decreasing returns to scale?

Problem 4

Consider a farmer with the following production function that uses labor L, capital K, and land N:

$$f(L, K, N) = L^{1/3}K^{1/6}N^{1/2}$$

This farmer faces costs for his input w for labor, r for capital, and p for land.

(a) What are the returns to scale for this production function?

(b) In the long run, the farmer can adjust all three inputs freely. Formally state the cost minimization problem if the farmer wants to produce a certain amount $\overline{y}$, and solve the long run cost minimization problem, finding the conditional demand for each input as well the long run minimum cost function.

(c) In the short run, the farmer has a fixed amount of land $\overline{N}$ that he cannot vary. Solve the short run cost minimization problem, again finding the conditional demand for each (free) input as well the short run minimum cost function.

(d) What input costs appear in the short run conditional demand functions? What input costs appear in the long run conditional demand functions? Explain in words why the two are either the same or different.

(e) Under what conditions will the short run minimum cost be the same as the long run minimum cost?

Problem 5

Consider a firm with the production function $f(L, K) = Kln(L)$. For the rest of this problem, assume that we are in the short run, so there is a fixed value of capital $\overline{K}$.

(a) What is the minimum (total) cost function?

(b) Say that $w = 2$, $r = 5$, and $\overline{K} = 2$. What are the fixed cost and variable costs, as functions of output?

(c) Draw total cost, fixed cost, and variable cost on the same graph.

(d) Find expressions for the AFC, AVC, and ATC in terms of output.

(e) Draw AFC, AVC, and ATC on the same graph.

(f) Find an expression for the marginal cost in terms of output.

(g) On a separate graph, draw the ATC, AVC, and MC. What do we know about the intersection of MC with each of the other two curves? Why is this the case (explain in words not math)?

Problem 6

You are the owner of three shoveling companies. Each one of these companies has a perfect complements technology that uses labor (L) and shovels (K) to produce the holes (y).

(a) The perfect complements technology can be expressed as: $f(K, L) = \min\{aK, bL\}$ for $a, b > 0$. For any two arbitrary levels of output y and y' ($y' > y$), draw the two isoquants explicitly showing the roles of a, b, y and y' on the two axes.

(b) More specifically, the technology faced by the three companies are given by:

$$f_1(K, L) = y_1 = [\min\{K, L\}]^{1/2}$$
$$f_2(K, L) = y_2 = \min\{K, L\}$$
$$f_3(K, L) = y_3 = [\min\{K, L\}]^2$$

For each one of the companies, explain whether it is facing a CRS, IRS or DRS.

(c) For simplicity, let $r = w = 1$. For each company, solve the cost minimization problem and find $K^*(y), L^*(y), C^*(y)$. (NB: we have removed r and w from the function's arguments as we have assigned numerical values to them, and for simplicity) (Hint: No need to use the Lagrangian method. Just use your economic intuition.)

(d) This part helps you understand how the returns to scale characteristic of the production function (from part b) relates to the shape of the cost function. Placing costs on the y-axis and output on the x-axis, draw the cost functions of each company from part c). You don't need to be precise but just capture the overall shape of the functions.

(e) Similarly, for each company, find the expression for the marginal cost and average total cost curves. Then, draw them (do them in three different graphs to avoid any confusion). In this part, we will learn how the shape of the cost function affects the shapes of the ATC and MC curves.