Econ 201 Section 5 - Problem Set 3

Due 2/16 by Start of Class - Graded for COMPLETION

Problem 1

Robinson derives utility from consumption of good (C) and leisure (R), and his utility function is $U(C, R) = \alpha logC + (1-\alpha)logR$ where $0 < \alpha < 1$. He is endowed with T hours of time that he can allocate either to leisure or to labor (L). Let p be the price of the good and w the per unit price of labor. Robinson also owns a firm that has access to a technology described by $f(l) = \beta\sqrt{l}$ where $\beta > 0$. In other words, he can produce the consumption good C using his own labor.

(a) By solving the profit maximization problem, find the firm's optimal choice of labor, $l^*(p, w)$. Also, find $\pi^*(p, w)$.

(b) Write down Robinson's time and budget constraints. Incorporate the time constraint into the budget constraint and rearrange to get full income (S) on one side.

(c) Using the final budget constraint from part (b), write down and solve Robinson's utility maximization problem. In other words, calculate $C^*(p, w, S)$, $R^*(p, w, S)$. Also write down $L^*(p, w, S)$. Assume that the SOCs are satisfied.

(d) To simplify the math, let $p = 1$. Calculate the equilibrium price (w^*) and quantities ($C^* = c^*, L^* = l^*$, and R^*)

(e) Let $p = 1$. How does equilibrium wage w^* change as α, β and T change? In other words, do some comparative statics by differentiating w^* in turn with respect to these three variables. Provide the economic intuition in each case.

(f) Let $p = 1$. How does equilibrium consumption C^* change as α, β and T change? Provide the economic intuition in each case.

(g) Let $p = 1$. How does equilibrium leisure R^* change as α, β and T change? Provide the economic intuition in each case.

(h) Robinson decides that setting up labor and good markets is complete nonsense. In the end, he is the only worker and he owns the only firm that will employ him. Plus, he is the only consumer out there. He still has access to the technology that turns his labor into consumption good. By solving his new optimization problem, find his optimal for C^*, R^* and L^*. Compare these results with the results from part (d). Justify your comparative statements.

Problem 2

This question concerns the Robinson Crusoe Economy where Robinson's utility function is given by $U(C,R) = C^{\alpha}R^{1-\alpha}$ for the consumption of good C and leisure R. He is endowed with T units of time that he can allocate either to leisure (R) or to labor (L). Let p be the price of the good and w the per unit price of labor. Robinson also owns a firm that has access to a technology described by $f(l) = Al$ where A is a constant.

(a) Draw the constraint of the optimization problem faced by Robinson on a graph with C on the y-axis and R on the x-axis. Make sure you label all relevant intercept/s and slope.

(b) Derive Robinson's optimal demands for C and R (and thus L) for any prices. You can assume that SOCs are satisfied.

(c) Derive the firm's profit maximizing demand for labor for any prices.

(d) Solve for the equilibrium prices and quantities of this economy.

(e) Thanks to technological progress the new production function is given by $f(l) = Bl$ where $B > A$. Explain how the equilibrium in the economy will change.

Problem 3

Suppose we have 100 identical consumers. Consumers' utility function is given by: $U(C,R) = C^{\alpha}R^{\beta}$ where C is the consumption good and R is leisure. Each consumer is endowed with 1 unit of time that can be allocated either to R or labor, L, at a wage rate w. In addition, each consumer is also endowed with 1 unit of capital that can rented at a rental rate r. Finally, each consumer has an equal share of the ownership of the firm (i.e., each consumer owns 1% of the firm). The firm has access to the following production function: $f(k,l) = k^{1/2}l^{1/2}$. We will normalize the price of the consumption good to $p = 1$.

(a) Solve the utility maximization problem for a representative consumer. Be very careful in writing the budget constraint as the consumer has many sources of income in this model.

(b) Set up the firm's profit maximization problem and find the FOCs. You can assume that the SOCs are satisfied.

(c) Using the above results, solve for the competitive equilibrium. In other words, find the equilibrium values for prices w^*, r^*, values for labor and capital (L^* and K^*), profits π^* and leisure R^*.

Problem 4

Suppose a monopolist faces the following market demand curve: $y(p) = 100 - p$.

(a) You are told that the monopolist has a production function with this property: $ATC = MC = 5$. Set up and solve the profit maximization problem for the monopolist. In other words, find the optimal output and profit levels.

(b) Let's consider a different production function for the monopolist. It is described by the following cost function: $TC(y) = 300 - 5y + \frac{1}{4}y^2$. Set up and solve the profit maximization problem for the monopolist. In other words, find the optimal output and profit levels.

(c) Let's draw the situation from part (b). In particular, draw the demand, marginal revenue, marginal cost and average total cost curves. Show the area that represents profits. (It doesn't have to be in scale or very precise. Just capture the key features of the situation)

(d) Is the solution in part (b) efficient? Justify your answer. Graphically (use the same graph from part c), show consumer surplus (CS), producer surplus (PS) and deadweight loss (DWL).

(e) At the output level where profits are maximized, find the price elasticity of demand. You know from class, that a profit maximizing monopolist should never operate on the inelastic portion of the demand curve. Confirm this result.

Problem 5

A pharmaceutical company has received a patent for its new cholesterol drug, essentially turning it into a monopolist in its market for the duration of the patent. The company has spent 1,000 to develop this drug (i.e., its fixed costs), but the variable cost of manufacturing the drug is negligible, $C(y) = 0$. The market demand curve is given by: $y(p) = 100 - p$.

(a) Write down the profit maximization problem for the pharmaceutical company. Let y be the variable of choice.

(b) Both graphically and algebraically, find the output level that maximizes the company's profits and the market price for the drug. On the graph, make sure you label all the intercepts. Indicate the area that represents consumer surplus and producer surplus. Are producer surplus and producer's profit the same in this case?

(c) Is the solution to part (b) efficient? Justify your answer. Both graphically (use the same graph from part (b)) and algebraically, find the deadweight loss (DWL), consumer surplus (CS) and producer surplus (PS).

(d) At the output level where profits are maximized, find the price elasticity of demand. You know from class, that a profit maximizing monopolist should never operate on the inelastic portion of the demand curve. Confirm this result.

Problem 6

Suppose a monopoly market has a demand function in which quantity demanded depends not only on market price (P) but also on the amount of advertising the firm does (A, measured in dollars). The specific form of this function is

$$Q = (20 - p)(1 + 0.1A - 0.01A^2)$$

The monopolistic firm's cost function is given by

$$TC = 10q + 15 + A$$

(a) Suppose there is no advertising ($A = 0$). What output will the profit-maximizing firm choose? What market price will this yield? What will be the monopoly's profits?

(b) Now let the firm also choose its optimal level of advertising expenditure. In this situation, what output level will be chosen? What price will this yield? What will the level of advertising be? What are the firm's profits in this case? [Hint: Part (b) can be worked out most easily by assuming the monopoly chooses the profit-maximizing price rather than quantity.]

Problem 7

Suppose a perfectly competitive industry can produce widgets at a constant marginal cost of \$10 per unit. Monopolized marginal costs rise to \$12 per unit because \$2 per unit must be paid to lobbyists to retain the widget producers' favored position. Suppose the market demand for widgets is given by

$$D = 1000 - 50P$$

(a) Calculate the perfectly competitive and monopoly outputs and prices.

(b) Calculate the total loss of consumer surplus from monopolization of widget production.

(c) Graph your results, indicate the area of surplus and explain how they differ from the usual analysis.