Econ 201 Section 5 - Problem Set 4

Due 2/28 by Start of Class - Graded for ACCURACY

Problem 1

Say that the city of Chicago grants you monopoly power in selling textbooks to college students. Your store serves two equally sized client groups - UChicago students and Northwestern students. Suppose the quantities demanded by these two groups are given by:

$$y^c = 10 - p^c$$
$$y^n = a - p^n$$

where $a < 10$. Assume that the marginal cost of textbook production is 0.

(a) Suppose you are restricted to setting one price. Calculate the price you should charge to maximize your profit as well as the quantity sold for any value of $a < 10$. Hint: There will be a kink in total demand at $p = a$ (because Northwestern students cannot demand a negative number of books). It's best to solve separately for the best prices above and below a.

(b) Suppose now that you can set two different prices to each group. Calculate the price charged and quantity sold to each group.

(c) How do the profits and quantities compare between parts a) and b). Explain the intuition for these results. Say that you gain the ability to read minds and can charge each student at their exact willingness to pay. How would profit and quantity in that situation compare to parts a) and b)? Why?

Problem 2

(a) Solve for all pure and mixed strategy Nash equilibria of the following two games (using any method you like). (Hint: the second game has infinitely many Nash equilibria.)

		Player 2	
		L	R
Player 1	T	$(6, 0)$	$(0, 6)$
	B	$(3, 2)$	$(6, 0)$

		Player 2	
		L	R
Player 1	T	$(0, 1)$	$(0, 2)$
	B	$(2, 2)$	$(0, 1)$

(b) In the following game, we will use both the concepts of "mixed strategy" and "strictly dominated strategy".

		Player 2		
		L	M	R
	T	$(6, 6)$	$(1, 2)$	$(3, 3)$
Player 1	C	$(2, 1)$	$(4, 7)$	$(4, 3)$
	B	$(3, 4)$	$(2, 5)$	$(3, 9)$

(i) For player 1, show that mixing two of his pure strategies strictly dominates the third pure strategy.

(ii) After eliminating the dominated strategy in part (i), show that for player 2 also mixing two pure strategies strictly dominates the third pure strategy. Eliminate it.

(iii) Find both the pure and mixed strategy NE for the residual game.

Problem 3

N bidders are bidding in an auction for one indivisible object. Bidder i has a private value $v_i < 100$ for the object. Each of them can placed a sealed bid (For simplicity, assume they can only bid in whole dollars and cannot bid over 100). The highest bid wins. Winner pays the second highest bid. Losers pay nothing and get nothing. If there are multiple winners, they evenly share the object and the payment.

(a) Who are the players in this game?

(b) What is the strategy set for each player? (i.e. What are the choices each player can choose from?)

(c) Describe the payoff function for each player. (It would be a function of the actions of all agents.)

(d) Define a Nash Equilibrium in this game. Note you do not need to actually solve for the NE - merely denote the circumstances that would be an NE for this game.

Problem 4

In class we mainly studied the static games where the action set for each agent includes finite discrete actions. Sometimes the action set can be infinite and continuous. Consider the following example. Two firms are selling to a market with a fixed market demand. Market price is given by the market demand

$$P(Q) = \begin{cases} a - bQ & \text{if } Q \leq \frac{a}{b} \\ 0 & \text{if } Q > \frac{a}{b} \end{cases}$$

where Q is the sum of the two firms' outputs. Each of them chooses a quantity to produce at a constant marginal cost $c < a$. Each firm wishes to maximize its own profits.

(a) Who are the agents in this game?

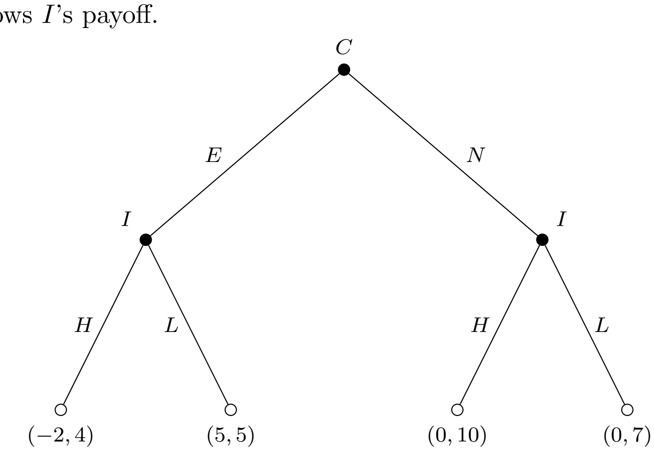
(b) What is the strategy set for each player? (i.e.What are the choices each player can choose from?)

(c) Describe the payoff function for each player. (It would be a function of the actions of all agents.)

(d) Define a Nash Equilibrium in this game. Note you do not need to actually solve for the NE - merely denote the circumstances that would be an NE for this game.

Problem 5

Consider two firms: an incumbent (I) and a potential competitor (C). First, the potential competitor has to decide whether to enter the market (E) or not enter the market (N), and then the incumbent has to decide whether to produce a high quantity (H) or low quantity (L). This game has the following extensive form. The first number in each payoff pair is C's payoff. The second number shows I's payoff.

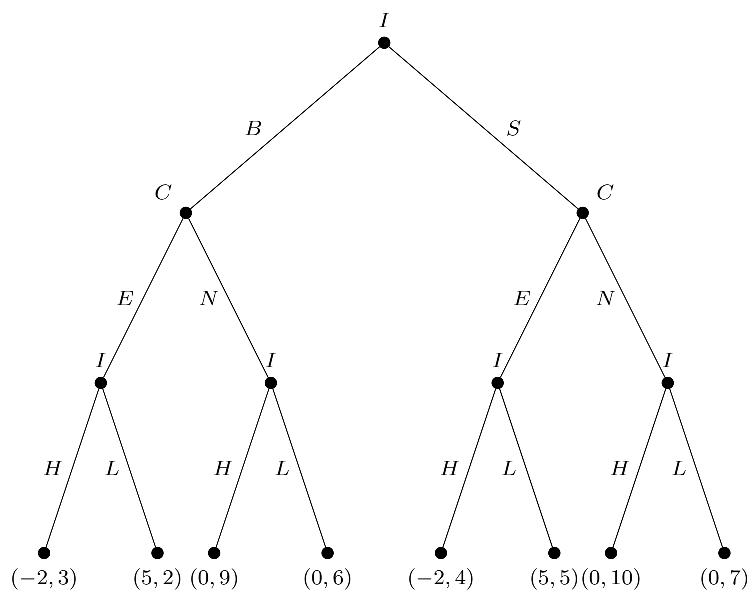


(a) Write down the set of strategies for each player, and the normal form of this game.

(b) Find the pure strategy Nash Equilibrium/a of this game.

(c) How many subgames does the main game have? What is/are the subgame perfect Nash equilibrium/a (SPNE)? Explain why some (if any) of the pure strategy NE are/is not SPNE.

(d) Consider the following change to the above game. Before the potential competitor makes a move, the incumbent can decide to build a big factory (B) or a small factory (S). The first number in each payoff pair is C's payoff. The second number shows I's payoff. The extensive form of the new game is:



In the above game, a strategy for C is written as (i, j) where i is the action when I plays B and j is the action when I plays S. Similarly, a strategy for I is written as $[v, (w, x), (y, z)]$ where v is the action in the initial node, (w, x) are the actions when C plays E and N respectively after I has played B (top branch), and (y, z) are the actions when C plays E and N respectively after I has played S (bottom branch). Using the above extensive form and backward induction, find all the SPNE (if any). Don't get confused with the payoffs.

Problem 6

Consider the following game. (A, A) is the only pure strategy Nash Equilibrium (NE) of this game. Although (B, B) is a Pareto improvement compared with the NE, it is not sustainable. Consider instead the case where this game is repeated an infinite number of times, and players both have a time discount factor β. Also, consider the following "threat": "I will play B as long as you play B. Once you play A, I will play A ever after." Find the condition for β so that (B, B) can be sustained as a NE given the threat. (Hint: Consider a deviation from (B, B) in the first period.)

		Player 2: A	Player 2: B
Player 1	A	$(-2, -2)$	$(0, -8)$
	B	$(-8, 0)$	$(-1, -1)$

Problem 7 - Bonus (5 pts on this PSet)

You and a friend have 1 brownie leftover after a party, and decide to split it. Specifically, you decide to split it via a sequential game - Player 1 cuts the brownie into two pieces and then Player 2 picks between the two pieces. Assume that both players' goal is to maximize the amount of brownie that they end up with (can define utility as the share of brownie received). Also assume that Player 1 is an extremely precise slicer - they can divide the brownie in exactly the manner they intend to.

(a) Describe the strategy sets of the two players.

(b) Find the NE for this game.

(c) Why would using this game be something that a Social Planner who values "fairness" like?