Econ 201 Section 5 - Problem Set 5

Due 3/7 by Start of Class - Graded for ACCURACY

Problem 1

Suppose there are two firms that sell newspapers in the market: ATN and PGM. The aggregate demand for newspapers in the market is given by

$$p = 12 - y_A - y_P$$

where y_A refers to ATN quantity and y_P refers to PGM quantity. Further suppose that the total cost functions for the two firms are given by:

$$TC^A(y_A) = y_A^2$$
$$TC^P(y_P) = \frac{1}{2}y_P^2$$

(a) Suppose the firms engage in Cournot competition. Find the Nash Equilibrium quantities produced by each firm, as well as the aggregate quantity of newspapers and each firm's profit.

(b) Suppose the two firms now consider a deal. In the deal, the two firms will agree to each produce an identical quantity of newspapers. This quantity will be set so as to maximize industry profits (the objective of the maximization problem will be overall profit). What will this quantity be set to?

(c) Will both firms be willing to agree to the deal? Why or why not? Does this make intuitive sense? Why or why not?

Problem 2

Consider the following Cournot model with three firms producing a homogenous good. Market demand is given by $Y = a - p$. Each firm faces a common constant marginal cost of c.

(a) What is the output and profit level of each firm?

(b) Suppose two of the firms merge. What is the output and profit level of each firm? Do the two firms have an incentive to merge?

Problem 3

Suppose there are two firms that sell houses in town - the Bluth Company and the Sitwell Company. The two firms sell differentiated houses - they are similar but not identical, and so act as substitutes in the market. Thus, the demand for either firm's houses will depend on the price set by the other firm. The demands and costs for the two products are:

$$y_b = 56 + 2p_s - 4p_b, \quad TC_b = 8y_b$$

$$y_s = 88 - 4p_s + 2p_b, \quad TC_s = 10y_s$$

(a) Suppose the two firms choose prices. What are their response functions?

(b) Solve for the equilibrium prices, quantities and profits for both firms.

Problem 4

In this question, we will formalize more the Bertrand model from class. We have two firms producing an identical good. Both firms have the same marginal cost $c > 0$ and they have no fixed costs. The firms compete by setting prices simultaneously (p_1 and p_2), and conditional on the chosen prices, quantity demanded from firm 1 is given by:

$$y_1(p_1, p_2) \begin{cases} 0 & \text{if } p_1 > p_2 \\ \frac{1}{2}\left(\frac{a-p_1}{b}\right) & \text{if } p_1 = p_2 \\ \frac{a-p_1}{b} & \text{if } p_1 < p_2 \end{cases}$$

Note that we would have a symmetric demand for firm 2 and when both firms charge the same price they equally split the market.

(a) Write down the set of strategies and payoffs for firm i.

(b) For a given $p_2 < a$, draw the demand curve faced for firm 1.

(c) For a given $p_2 < a$, draw the payoff function for firm 1 (i.e., place π, payoff, on the y-axis and p on the x-axis).

(d) Find the Nash equilibrium of this game by showing that the following cases are not a NE: (i) $p_1 > p_2 > c$, (ii) $p_1 > p_2, p_2 < c$, (iii) $p_1 = p_2 > c$, and (iv) $p_1 = p_2 < c$.

Problem 5

We have two firms in the market. Firm 1 is the leader and chooses its quantity first (y_1). Firm 2 is the follower and chooses its quantity (y_2) after observing Firm 1's choice. The inverse demand function is: $p(Y) = a - bY$, where $Y = y_1 + y_2$. The firms have the following cost functions: $C_1(y_1) = c_1 y_1$ and $C_2(y_2) = c_2 y_2$.

(a) Using backward induction, solve for the equilibrium quantities of this game (i.e., y_1^{S*} and y_2^{S*}). You can assume that the SOCs are satisfied.

(b) Find Y^*, p^{S*}, π_1^{S*}, and π_2^{S*}.

Problem 6

Suppose there are n firms, each with cost $c(y) = y$, playing the repeated Cournot quantity setting game. All firms discount future periods at rate $\beta \in (0, 1)$. Assume the inverse market demand is $p(Y) = a - bY$. Use Y^m to denote the monopoly quantity. Consider the following profile of strategies: Each firm sets quantity $y_i = \frac{Y^m}{n}$ in the first period, and continue setting this quantity as long as nobody has deviated from it in the past. If any firm has deviated from this strategy, all firms play the Cournot Nash Equilibrium quantity in all future periods. We are interested in knowing when this profile of strategies constitute a Nash Equilibrium.

(a) What are the profits of each firm in the current period if all firms choose the Cournot Nash Equilibrium quantities?

(b) If the firms collude to maximize collective profits in the current period, what are the profits of each firm?

(c) If the other $n - 1$ firms in the market were all choosing the collusive level of output, what quantity of output maximizes the profits of the nth firm in the current period? What are the profits this firm would receive in the current period?

(d) Based on your answers to part (a)–(c), determine the level of patience (β) necessary to support the above strategies as a Nash Equilibrium.

(e) How does your answer to part (d) vary with n? What does this tell you about how easy or difficult it is to support collusion when the number of firms increases?