Econ 201 Section 5 - Problem Set 6

Due 3/16 by 12:30 PM - Graded for COMPLETION

Problem 1

A fruit farmer produces fruits (F), and in the process uses a natural fertilizer (a). The farmer's cost function is $TC_F(F, a) = 0.5F^2 + (a-2)^2$. Although humans cannot taste the fertilizer, bees are attracted to it, and the more fertilizer the farmer uses, the more honey the bees can produce for the honey producer. The cost function for the honey producer is given by: $TC_H(H, a) = 0.5H^2 + (2-a)H$. For simplicity, assume that $p_F = p_H = 1$.

(a) Solve the private profit maximization problem for the fruit farmer and calculate its profits. You can assume that SOCs are satisfied.

(b) Solve the private profit maximization problem for the honey producer and calculate its profits. You can assume that the SOC is satisfied.

(c) Solve for the socially efficient solution. In other words, assume that a social planner owns both the fruit farm and honey business. What's the joint profit level?

(d) Explain why your results from a)-b) are identical/different to those from c).

Problem 2

We present here a negative externality problem that is framed a bit differently compared to the one we did in class, but we can still use the same analytical steps. A natural gas company is closely located near a lake where a boat rental store is also located. The gas company faces the following inverse demand curve: $p_g = 200 - g$. The plant's cost function is $C_g = 2g^2$. Similarly, the boat rental store faces the following inverse demand function: $p_B = 100 - B - 2g$. In words, as the gas company produces more g, the demand for boat rentals decreases (because pollution makes a boat ride less pleasant). The store's cost function is given by: $C_B = 0.5B^2$. Throughout this question, you can assume that SOCs are satisfied.

(a) Solve the private profit maximization problems for the gas company and the boat rental. For both businesses, calculate the optimal quantities, prices and profit levels.

(b) Solve for the socially efficient solution. In other words, assume that a social planner owns both businesses. Similar to part a), find quantities, prices, and the joint profit level. Specify how much each business contributes towards joint profits.

(c) Compare the firm/joint profit levels between parts a) and b).

(d) You work for the environmental agency regulating pollution in the lake. If you want to implement a quota, on which business would you impose the quota, and at what level would you set it? In terms of quantities, prices and profit levels, for both firms, what are the new equilibrium levels? (Hint: you need no further calculations)

(e) Instead, consider now imposing a Pigouvian tax on the steel plant. Solve the new profit maximization problem for the gas company, and find the equilibrium tax rate T^* that achieves the socially efficient outcome. (Hint: different to what we did in class, the tax rate T is imposed on units of g)

(f) Finally, let's consider implementing property rights and trade. As the government agency you decide that the boat rental store has the right for a clean lake. Furthermore, you create a 'market' where the gas company can buy the 'right' to pollute from the boat rental store. Given that pollution is an unavoidable byproduct of producing g, essentially the gas company will be paying p_{gT} for the right to produce one unit of g. Without using any shortcuts, and by solving the profit maximization problems for both businesses, find p^*_{gT} and confirm that the equilibrium quantities and prices for both businesses are identical to those from the socially efficient solution.

Problem 3

This question analyses public goods in the context of firms (in contrast to consumers we discussed in class). Two hot dog stands (H1 and H2) are located in the same courtyard of a mall. They can invest in flowers (the public good) to beautify the courtyard: $F = F_1 + F_2$. Once the flowers are planted, both hot dog stands benefit from the beautified courtyard through increased profits from hot dog sales. Their profit functions are: $\pi_1 = 4ln(H_1) - H_1 + 2ln(F) - F_1$ and $\pi_2 = 4ln(H_2) - H_2 + ln(F) - F_2$, where H_1 and H_2 are the number of hot dogs sold by the corresponding stands. Throughout the question, assume that SOCs are satisfied.

(a) Assume that H2 decides to invest $F_2 = 0.5$. If H1 wants to maximize its profits, how much should it invest in F_1?

(b) Note that in the current setup, the two hot dog stands are playing a Cournot game simultaneously deciding how much to invest in flowers. So we can use the concept of best response functions to solve for the

equilibrium. Formally, set up the profit maximization problems for H1 and H2, and find their best response functions for investments in flowers.

(c) Now let's solve for the equilibrium, and this is the case we skipped in class. Solve simultaneously the best response functions from part b) to find the equilibrium. Were you possible to find the equilibrium mathematically?

(d) This is a case of a corner solution, and the graphical representation will aid in finding the equilibrium. By placing F_1 on the horizontal axis, and F_2 on the vertical axis, draw the two best response functions. In particular, when drawing $F_1^*(F_2)$ be very precise what happens when $F_2 \geq 2$. And when drawing $F_2^*(F_1)$ be very precise what happens when $F_1 \geq 1$. Find the equilibrium and provide the economic intuition behind it (i.e., in terms of free riding).

(e) By solving the joint profit maximization problem, find the socially efficient level of investment in flowers (it's ok to just find the FOC only for F).

Problem 4

Consider a very simplified world with two individuals: 1 and 2. The two individuals have the same income: $m_1 = m_2 = 100$. Agent $i (i = 1, 2)$ can spend his endowment on two goods: a private good C_i and on education E_i, a public good. Thus, $E = E_1 + E_2$. The two individuals have identical utility function: $U_i(C_i, E) = 5ln(C_i) + 2ln(E)$. The prices of the goods are: $p_C = 2$ and $p_E = 3$. Throughout the question, assume that SOCs are satisfied.

(a) By setting up and solving the utility maximization problem of the two individuals, calculate how much each individual will be spending on education and the private good. (Hint: exploit the symmetry of the problem between the two individuals to simplify your work)

(b) Using the same procedure as in class, find the socially efficient level of education and the individual consumption levels of the private good.

(c) Compare total investments in education from parts a) and b). Are the individuals better off in part a) or part b)? Justify your answer.