Econ 21020 - Problem Set 1

Due 10/11 at 11:59 PM. Submit to Canvas. May be completed in groups of up to 6 students (if the group members are made known to the instructor prior to submission). Only one submission is required per group.

Problem 1

(a) Express the probability mass function and cumulative density function for a balanced 6-sided die (numbered 1-6) and draw images representing the functions (the images need not be extremely precise - they just need to capture the important attributes of the functions).

(b) Calculate the expectation and variance of the outcome of a balanced 6-sided die roll. Calculate the expectation of the square of the outcome of a balanced 6-sided die roll (i.e. $E[X^2]$ if X represents the outcome of the roll).

(c) In class, we saw the CDF of a Uniform[a,b] variable. Demonstrate how this CDF could be derived from the PDF we saw (recreated below) and explain the two functions' relationship in words.

$$f(x) = \begin{cases} \frac{1}{b-a} & if\ a \leq x \leq b \\ 0 & if\ otherwise \end{cases}$$

Problem 2

Say that n is some counting number (1,2,3,...) and that there are a collection of independent Bernoulli random variables, $(X_1, X_2, X_3, ..., X_n)$, each of which is equal to 1 with probability p and equal to 0 with probability $1 - p$ (all n variables have identical distributions). Define $X^* = \sum_{i=1}^{n} X_i$.

(a) Show that $E[X^*] = np$. Justify all steps.

(b) Show that $Var(X^*) = np(1 - p)$. Justify all steps.

Problem 3

The following table represents the joint probability mass function of college graduation status and employment status among the working-age population of South Africa.

	Unemployed (Y=0)	Employed (Y=1)
Non-college grads (X=0)	0.078	0.673
College grads (X=1)	0.042	0.207

(a) Explain in words what the number 0.078 in the top-left cell means.

(b) Calculate the marginal probability of being unemployed ($P(Y = 0)$) and the marginal probability of being a college graduate ($P(X = 1)$).

(c) Calculate the likelihood of unemployment both for non-college grads and for college grads ($P(Y = 0|X = 0)$ and $P(Y = 0|X = 1)$).

(d) Are college graduation status and employment status independent? Are they mean independent? Demonstrate that they are or are not.

Problem 4

Prove each of the following statements using the results on the slide "Properties of Expectations" and the two given definitions of covariance and two given definitions of variance. Justify all steps.

(a) $\text{Cov}(X, a) = 0$

(b) $\text{Cov}(X + Y, Z) = \text{Cov}(X, Z) + \text{Cov}(Y, Z)$

(c) $\text{Cov}(a + bX, Y) = b\text{Cov}(X, Y)$

(d) $Var(X + Y) = Var(X) + Var(Y) + 2Cov(X, Y)$

Problem 5

Prove that, for conditional variance, it is true that,

$$\text{Var}[g(X) + h(X)Y|X] = h^2(X)\text{Var}[Y|X]$$

for functions $g()$ and $h()$ and random variables X and Y using the definition of conditional variance and properties of conditional expectations. Justify all steps.

Problem 6

(a) Give an example of two random variables that are uncorrelated but not mean independent (different from any examples given in the notes) and show that the former property holds and the latter does not.

(b) Give an example of two random variables that are mean independent but not independent (different from any examples given in the notes) and show that the former property holds and the latter does not.

Problem 7

Perform the following using R or another statistical software of your choice (provided that you have cleared any alternative option with the TA).

(a) Generate 1,000 draws from a standard normal distribution ($N(0, 1)$) and plot the simulated data in a histogram.

(b) Generate 1,000 draws from a Uniform$[-1, 1]$ distribution and plot the simulated data in a histogram.