Econ 201 Section 5 - Problem Set 2

Due 2/2 by Start of Class - Graded for ACCURACY

Problem 1

Consider a firm with the production function $f(K, L) = L^{1/3}K^{1/3}$

(a) Solve the cost minimization problem for this firm, for any given value of output y (Step 1 of Indirect Method of profit maximization).

SOLUTION: Cost minimization problem:

$$\min_{L,K} wL + rK$$

$$s.t.\ \overline{y} = f(L, K) = L^{1/3}K^{1/3}$$

$$\mathcal{L} = wL + rK + \lambda(\overline{y} - L^{1/3}K^{1/3})$$

$$[L]:\ w = \lambda\frac{1}{3}L^{-2/3}K^{1/3}$$

$$[K]:\ r = \lambda\frac{1}{3}L^{1/3}K^{-2/3}$$

$$[\lambda]:\ \overline{y} = L^{1/3}K^{1/3}$$

Combine the first and second FOCs to get:

$$\frac{w}{r} = \frac{K}{L}$$

$$L = \frac{r}{w}K$$

Now plug this into the third FOC:

$$\overline{y} = (\frac{r}{w}K)^{1/3}K^{1/3}$$

$$K^*(w, r, \overline{y}) = (\frac{w}{r})^{1/2}\overline{y}^{3/2}$$

A symmetrical process will yield conditional labor demand, which we can plug, along with conditional capital demand from above, into a minimum cost function:

$$L^*(w, r, \overline{y}) = (\frac{r}{w})^{1/2}\overline{y}^{3/2}$$
$$C^*(w, r, \overline{y}) = w(\frac{r}{w})^{1/2}\overline{y}^{3/2} + r(\frac{w}{r})^{1/2}\overline{y}^{3/2}$$
$$C^*(w, r, \overline{y}) = 2(wr)^{1/2}\overline{y}^{3/2}$$

(b) Using your result from part a), find the profit maximizing value of output (Step 2 of Indirect Method of profit maximization). Find the optimal amount of profit.

SOLUTION: Profit maximization problem:

$$\max_y py - C^*(w, r, y)$$

$$\max_y py - 2(wr)^{1/2}y^{3/2}$$

$$[y]: \; p = 3(wr)^{1/2}y^{1/2}$$
$$y^*(w, r, p) = (\frac{p}{3})^2(wr)^{-1}$$

Now we can plug this into our profit function to get optimal profit from the indirect method:

$$\pi = py^* - 2(wr)^{1/2}(y^*)^{3/2}$$
$$\pi = p(\frac{p}{3})^2(wr)^{-1} - 2(wr)^{1/2}((\frac{p}{3})^2(wr)^{-1})^{3/2}$$
$$\pi^*(w, r, p) = \frac{1}{27}\frac{p^3}{wr}$$

(c) Now, set up and solve the profit maximization problem for this firm using the Direct Method of profit maximization.

SOLUTION: Direct profit maximization problem:

$$\max_{L,K} pf(L, K) - wL - rK$$

$$\max_{L,K} pL^{1/3}K^{1/3} - wL - rK$$

$$[L]: \ w = \frac{p}{3}L^{-2/3}K^{1/3}$$
$$[K]: \ r = \frac{p}{3}L^{1/3}K^{-2/3}$$

Combine the FOCs to get:

$$\frac{w}{r} = \frac{K}{L}$$
$$L = \frac{r}{w}K$$

Plug this into the second FOC to get:

$$r = \frac{p}{3}(\frac{r}{w}K)^{1/3}K^{-2/3}$$
$$\hat{K}(w,r,p) = \frac{p^3}{27}r^{-2}w^{-1}$$

A symmetric argument will give us the unconditional demand for labor. We can then plug this, along with the unconditional demand for capital, into the profit function to find the optimal level of profit:

$$\hat{L}(w,r,p) = \frac{p^3}{27}r^{-1}w^{-2}$$
$$\hat{\pi}(w,r,p) = (\frac{p^3}{27}r^{-1}w^{-2})^{1/3}(\frac{p^3}{27}r^{-2}w^{-1})^{1/3} - w\frac{p^3}{27}r^{-1}w^{-2} - r\frac{p^3}{27}r^{-2}w^{-1}$$
$$\hat{\pi}(w,r,p) = \frac{1}{27}\frac{p^3}{wr}$$

(d) Do the results from the Indirect and Direct methods agree with one another? Explain why we would expect this is the case.

SOLUTION: We see that we get the same final amount of profit regardless of which method we use. This is to be expected - if we got different answers for the two methods this would suggest that we were not actually optimizing for at least one of the methods. If the final profit yielded by the two methods is unequal, one of those two profits is less than the other. But then, then the method that yielded the lesser profit was not optimizing - we could do better via the other method. Thus, if both methods are actually optimizing profit, and if both face the same technology and prices, they should yield the same final profits.

Problem 2

Continue to work with your results from the first question.

(a) Demonstrate that the unconditional input demands are homogeneous of degree 0. Explain the intuition for this result.

SOLUTION:

$$\begin{aligned}
\hat{K}(tw, tr, tp) &= \frac{t^3 p^3}{27} t^{-2} r^{-2} t^{-1} w^{-1} \\
\hat{K}(tw, tr, tp) &= \frac{p^3}{27} r^{-2} w^{-1} \\
\hat{K}(tw, tr, tp) &= \hat{K}(w, r, p) \\
\hat{L}(tw, tr, tp) &= \frac{t^3 p^3}{27} t^{-1} r^{-1} t^{-2} w^{-2} \\
\hat{L}(tw, tr, tp) &= \frac{p^3}{27} r^{-1} w^{-2} \\
\hat{L}(tw, tr, tp) &= \hat{L}(w, r, p)
\end{aligned}$$

This result holds because only relative prices matter in the optimal production decision. The FOCs of the profit maximization problem tell us that we want to equate the cost of a unit of input and the value created by a unit of output. If we scale both of those up by the same amount, by scaling up the price of each input as well as output, then that equation will remain true, and the firm has no need to shift from the point it was operating at.

(b) Demonstrate that the unconditional input demands will decrease in the prices of their own factors. Explain the intuition for this result.

SOLUTION:

$$\begin{aligned}
\frac{d}{dw}\hat{L}(w, r, p) &= \frac{d}{dw}\frac{p^3}{27} r^{-1} w^{-2} \\
&= \frac{-2p^3}{27} r^{-1} w^{-3} \leq 0 \; \forall \; (w, r, p) \geq 0 \\
\frac{d}{dr}\hat{K}(w, r, p) &= \frac{d}{dr}\frac{p^3}{27} r^{-2} w^{-1} \\
&= \frac{-2p^3}{27} r^{-3} w^{-1} \leq 0 \; \forall \; (w, r, p) \geq 0
\end{aligned}$$

We see that unconditional input demands fall in their own prices. This is to be expected for two reasons. First, increasing the price of one input while keeping the price of the other input and amount of output constant will mean that we will likely want to shift towards the other input. Increasing the price of one input will decrease the production per dollar of that input, meaning that the cost minimization FOCs would no longer hold, and we will want to shift towards the other, now more relatively cost efficient, input. This is the substitution effect. Unconditional input demands also endogenously reflect our choice of how much total output to produce. If one input price rises, cost of production will likely generally rise. This would equate to increasing the marginal cost, meaning that (for constant price) we will want to decrease total production, thereby lowering the required amount of both inputs. This is the scale effect.

(c) Demonstrate that optimal profit is homogeneous of degree 1. Explain the intuition for this result.

SOLUTION:

$$\hat{\pi}(tw, tr, tp) = \frac{1}{27}\frac{t^3p^3}{twtr}$$
$$\hat{\pi}(tw, tr, tp) = \frac{1}{27}\frac{tp^3}{wr}$$
$$\hat{\pi}(tw, tr, tp) = t\hat{\pi}(w, r, p)$$

From the answer to part a), we know that scaling up all prices by the same amount will not result in the firm changing its production or amounts of inputs. However, due to the scaling up in prices, the amount of revenue and amount of cost will both scale up by the same amount, which means the amount of profit scales up by that same amount. Intuitively, if we just changed the units we are measuring profits in, this won't change how we produce, but we will report our "same old" profit in the new units.

(d) Find the derivative of optimal profit with respect to each input price. Explain the intuition for this result.

SOLUTION:

$$\begin{aligned}\frac{d\hat{\pi}(w,r,p)}{dw} &= \frac{d}{dw}\frac{1}{27}p^3(wr)^{-1} \\ &= \frac{-1}{27}p^3w^{-2}r^{-1} \\ \frac{d\hat{\pi}(w,r,p)}{dr} &= \frac{d}{dr}\frac{1}{27}p^3(wr)^{-1} \\ &= \frac{-1}{27}p^3w^{-1}r^{-2}\end{aligned}$$

We see that both derivatives are negative. This is what we would expect, as increasing the cost of one input will generally make production more expensive, increasing cost, while adding no value to revenue. This should tend to decrease profit. Put another way, we know that firms will change output in response to an isolated change in one factor's price, from part b). However, if the firm was at the optimum before, then a change in the amount of production must imply a decrease in profits.

(e) (Bonus - 3 pts on this assignment) Find a way to decompose the derivative of the unconditional input demands into a substitution and scale effect. Is there any way of knowing which is larger?

SOLUTION: From the lecture notes, we got that the substitution and scale effects were derived from the observation that the unconditional input demands were the same as the conditional input demands evaluated at the profit maximizing value of output:

$$\begin{aligned}
\hat{L}(w,r,p) &= L^*(w,r,y^*(w,r,p)) \\
\Rightarrow \frac{d}{dw}\hat{L}(w,r,p) &= \frac{d}{dw}L^*(w,r,y^*(w,r,p))
\end{aligned}$$

Thus, we can find the scale and substitution effects for our context by taking the derivative of the conditional labor demand with the optimal output inputted in (note that we should avoid simplifying the expression too much so that we can apply the product rule):

$$\begin{aligned}
L^*(w,r,y^*(w,r,p)) &= (\frac{r}{w})^{1/2}(y^*)^{3/2} \\
&= (\frac{r}{w})^{1/2}((\frac{p}{3})^2(wr)^{-1})^{3/2} \\
\frac{d}{dw}L^*(w,r,y^*(w,r,p)) &= \frac{d}{dw}(\frac{r}{w})^{1/2}((\frac{p}{3})^2(wr)^{-1})^{3/2} \\
&= -\frac{1}{2}r^{1/2}w^{-3/2}(\frac{p}{3})^3(wr)^{-3/2} - \frac{3}{2}r^{1/2}w^{-1/2}(\frac{p}{3})^3w^{-5/2}r^{-3/2} \\
&= -\frac{1}{2}r^{-1}w^{-3}(\frac{p}{3})^3 - \frac{3}{2}r^{-1}w^{-3}(\frac{p}{3})^3 \\
&= (-\frac{1}{2}-\frac{3}{2})r^{-1}w^{-3}(\frac{p}{3})^3
\end{aligned}$$

Thus, we can see that we have two nearly identical terms, with a different leading coefficient. Following the logic from the lecture notes, the first term (with the $-\frac{1}{2}$ out front) is the substitution effect (derivative with respect to the the w "from" the conditional labor demand) and the second term (with the $-\frac{3}{2}$ out front) is the scale effect (derivative with respect to the the w "from" the optimal value of output). With all non-negative prices, this indicates that the scale effect will always dominate, i.e. have a larger absolute value in this setting. We can also confirm that the above expression is the same as taking the derivative of the unconditional labor demand with respect to wage directly:

$$\frac{d}{dw}\hat{L}(w, r, p) = -2r^{-1}w^{-3}(\frac{p}{3})^3$$

Problem 3

Consider a firm operating in perfect competition with a total cost of $TC(y) = y^3 + 20$ in the short run.

(a) Express the profit function for this firm.

SOLUTION: As usual, profit is equal to total revenue (price times cost for a competitive firm) minus total cost:

$$\pi(y) = py - y^3 - 20$$

(b) What will be the short-run profit maximizing amount of output for this firm if it faces a price of 12?

SOLUTION: For a competitive firm, we will want to, first, find the value of output that will equal marginal cost and price, and then verify that the firm will choose that level of output over shutting down.

$$\begin{aligned} MC(y) &= \frac{dTC(y)}{dy} \\ &= \frac{d}{dy}(y^3 + 20) \\ &= 3y^2 \\ 12 = 3y^2 &\Rightarrow y^{SR} = 2 \end{aligned}$$

We now verify that operating at this point is better than shutting down. In the short run, as we do not need to consider fixed costs, this is equivalent to checking that MC exceeds AVC.

$$\begin{aligned} AVC(y) &= \frac{VC(y)}{y} \\ &= \frac{y^3}{y} \\ &= y^2 \\ 3y^2 > y^2 \; \forall \, y > 0 &\Rightarrow MC(y) > AVC(y) \; \forall \, y > 0 \end{aligned}$$

Thus, the firm will want to operate, and 2 is indeed the correct answer.

(c) Suppose we move into the long run. In the long run, the firm only pays the $20 in fixed cost if it produces any amount of output. (If it produces 0 output, its fixed cost will now be 0.) Does this change the firm's decision on how much to produce?

SOLUTION: If the firm does produce in the long run, it will still operate at the same point implied by $MC = p$, which was 2. However, now that we know the fixed cost will only be paid if the firm operates, the question of whether the firm wants to operate at all is a question of whether MC exceeds ATC, aka whether the firm earns positive profit from operating:

$$\begin{aligned}
ATC(y) &= \frac{TC(y)}{y} \\
&= \frac{y^3 + 20}{y} \\
&= y^2 + \frac{20}{y} \\
MC(2) &\lesseqgtr ATC(2) \\
3(2)^2 &\lesseqgtr 2^2 + \frac{20}{2} \\
12 &< 14
\end{aligned}$$

We see that putting in the fixed cost means that the firm will lose money if they operate. Thus, the firm should go out of business, and produce 0 output.

(d) Is there a minimum price at which the firm chooses to produce in the short run? In the long run?

SOLUTION: We saw in part b) that $MC > AVC$ for all positive values of output. The fact that $MC(y) = 3y^2$ also means that any positive price will imply a positive amount of production. Thus, in the short run, the firm will want to operate at any price, in the short run.

In part c), we saw that there is at least one price at which the firm will not want to operate in the long run. Thus, we need to check if there is a minimum price that will induce $MC > ATC$:

$$\begin{aligned}
MC(y) &> ATC(y) \\
3y^2 &> y^2 + \frac{20}{y} \\
y &> 10^{1/3}
\end{aligned}$$

Thus, we have found that $10^{1/3}$ is the minimum, profitable value of output. We can plug this into MC to find the price that will induce this level of output, and this will give us the minimum price at which the firm will operate in the long run.

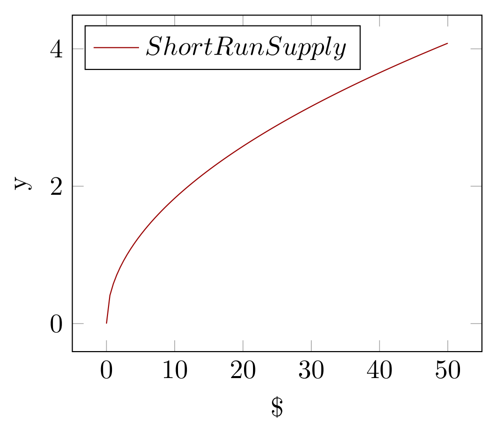
$$MC(10^{1/3}) = 3(10)^{2/3}$$

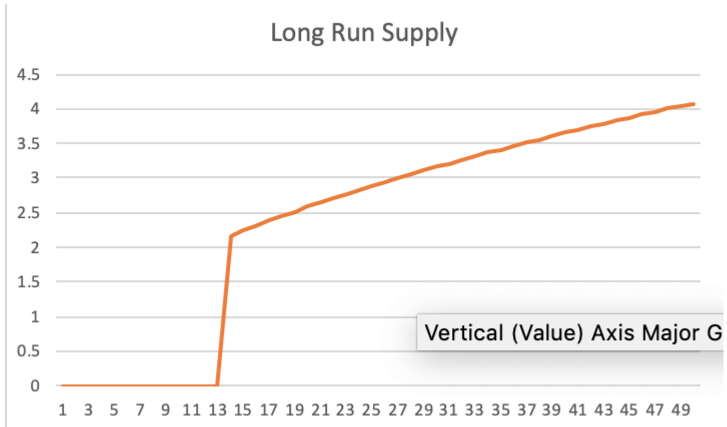
(e) Find expressions for the short-run and long-run supply curves for this firm. Graph them (no need to be very exact - just capture the general shape).

SOLUTION: From the previous part, the short-run supply will simply be the inverse MC, for any value of price. For the long run, the supply will be the same, for any value greater than $3(10)^{2/3}$ and 0 before that:

$$\begin{aligned} p &= MC(y^{SR}) \\ y^{SR} &= MC^{-1}(p) \\ y^{SR} &= (\frac{p}{3})^{1/2} \\ y^{LR} &= \begin{cases} (\frac{p}{3})^{1/2} \; if \; p > 3(10)^{2/3} \\ 0 \; if \; p \leq 3(10)^{2/3} \end{cases} \end{aligned}$$

The below graphs show short- and long-run supplies. Note that our typical graphs show inverse supply, so the axes here are the reverse of what is typical. Showing inverse supply would convey the same exact information, and would simply mean flipping the axes back to "normal."





Problem 4

Consider a representative consumer with a utility function over two goods (x, y) given by $U(x, y) = x^{1/3}y^{2/3}$ and let M denote her income. On the supply side of good y, the cost function of a representative firm is given by $C(y) = \frac{1}{2}y^2$. Let p_x and p_y be the corresponding prices of the two goods.

(a) Find the consumer's demand for good y. In other words, find $y^D(p_y)$ while holding p_x and M constant.

SOLUTION: Consumers choose demand so as to maximize utility while subject to a budget constraint. We can solve using a Lagrangian:

$$\mathcal{L} = x^{1/3}y^{2/3} + \lambda(M - p_x x - p_y y)$$

$$[x] : \frac{1}{3}x^{-2/3}y^{2/3} = \lambda p_x$$

$$[y] : \frac{2}{3}x^{1/3}y^{-1/3} = \lambda p_x$$

$$[\lambda] : M = p_x x + p_y y$$

We can combine our first two FOCs to say:

$$\frac{y}{2x} = \frac{p_x}{p_y}$$

and then sub in with the constraint to get demand for y:

$$y^D(p_y) = \frac{2M}{3p_y}$$

(b) By solving the profit maximization problem, find the firm's supply curve, $y^S(p_y)$.

SOLUTION: Firms will maximize profit:

$$\max_y p_y y - \frac{1}{2}y^2$$

This yields an FOC:

$$p_y - y = 0$$

and we can see that the SOC is always satisfied:

$$-1 < 0$$

So our supply curve is simply:

$$y^S(p_y) = p_y$$

(c) Calculate the partial equilibrium price p_y^* and quantity y^* in the market for good y where you only have one representative consumer and one firm (the firm acts as it would under perfect competition).

SOLUTION: The equilibrium is composed of the price and quantity where the markets clear. Thus, we set our supply and demand equal to one another:

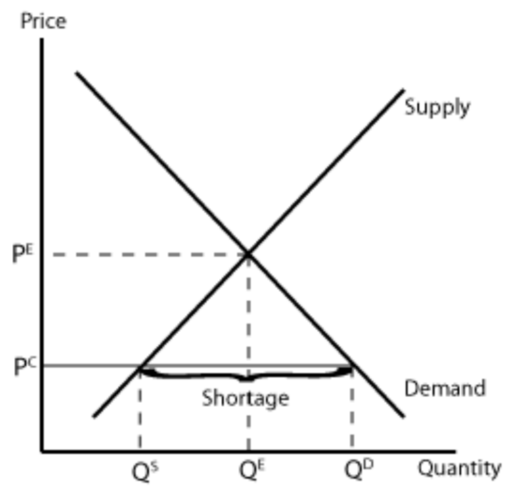
$$y^S(p_y) = y^D(p_y)$$
$$p_y = \frac{2M}{3p_y}$$
$$p_y^* = (\frac{2M}{3})^{1/2}$$

We then plug equilibrium price into either supply or demand to get equilibrium quantity:

$$y^* = (\frac{2M}{3})^{1/2}$$

(d) The government is not happy with the equilibrium price and decides to set a price ceiling p_y^C (i) Should p_y^C be set above or below p_y^*? Justify your answer. (ii) Draw the graph of the market for good y with the price ceiling. You do not have to be precise but capture the main features of the demand and supply curves and the price ceiling. (iii) Find the quantity that will be traded in the market and the profit level of the firm. (iv) Find also the utility level of the consumer with the price ceiling.

SOLUTION:
(i) If the government wants the price ceiling to have any effect, it needs to be below the equilibrium price. If $p_y^C \geq p_y^*$, then the market will not change, as it is already operating at a legally compliant point.
(ii)



(iii) If the price ceiling is set below equilibrium price, it will bind and so quantity will be where this price intersects supply. The consumers will demand more, but there is no one to sell it to them, so the amount traded in the market is simply:

$$y^S(p_y^C) = p_y^C$$

We can find profit by plugging this amount into our firm profit function:

$$\begin{aligned}\pi(p_y^C) &= p_y^C p_y^C - \frac{1}{2}(p_y^C)^2 \\ &= \frac{1}{2}(p_y^C)^2\end{aligned}$$

(iv) The consumer will consume exactly p_y^C, per the discussion of the previous part. They will then spend the rest of their budget on good x (as they have no other options, and will strictly prefer this to letting that money go to waste). We can plug this into the consumer's utility function to get:

$$U(\frac{M - (p_y^C)^2}{p_x}, p_y^C) = (\frac{M - (p_y^C)^2}{p_x})^{1/3}(p_y^C)^{2/3}$$

(e) By comparing the profit levels before and after the intervention, show whether the profits have increased or decreased.

SOLUTION: We can plug in the original equilibrium quantity into the firm's profit function to get the original level of profit:

$$\pi(p_y) = p_y^* p_y^* - \frac{1}{2}(p_y^*)^2$$
$$= \frac{1}{2}(p_y^*)^2$$

Because the government set it so that $p_y^C < p_y^*$, this indicates that profit fell as a result of the price ceiling.

Problem 5

The handmade snuffbox industry is composed of 100 identical firms, each having short-run total costs given by $STC = 0.5q^2 + 10q + 5$ where q is the output of snuffboxes per day.

(a) What is the short-run supply curve for each snuffbox maker? What is the short-run supply curve for the market as a whole?

SOLUTION: We know that firms will supply at $MC = p$, so long as $MC > AVC$:

$$\begin{aligned} MC(q) &= \frac{dVC(q)}{dq} \\ &= \frac{d}{dq}(0.5q^2 + 10q) \\ &= q + 10 \\ AVC(q) &= \frac{VC(q)}{q} \\ &= \frac{0.5q^2 + 10q}{q} \\ &= 0.5q + 10 \end{aligned}$$

We see that $MC > AVC$ everywhere. However, we need $p \geq 10$ for $MC = p$ to imply a positive amount of production. Thus, our short run supply for each individual firm will be:

$$q_i^{SR} = \begin{cases} p - 10 \; if \; p \geq 10 \\ 0 \; if \; p < 10 \end{cases}$$

As all 100 firms are identical, all 100 will have the same individual supply. Thus, to get market supply, we simply need to multiply the above by 100:

$$q_{tot}^{SR} = \begin{cases} 100p - 1000 \; if \; p \geq 10 \\ 0 \; if \; p < 10 \end{cases}$$

(b) Suppose the demand for total snuffbox production is given by $Q = 1100 - 50P$. What will be the equilibrium in this marketplace? What will each firm's total short-run profits be?

SOLUTION: Equilibrium will occur at the price and quantity such that market supply and demand equal one another. Assume that we will have positive supply, so:

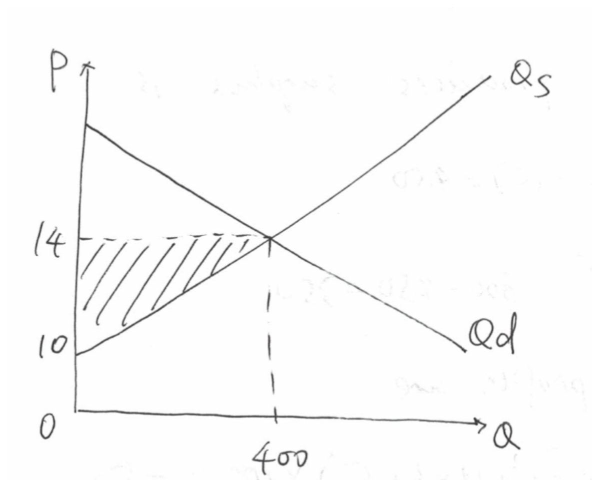
$$q_{tot}^{SR} = Q$$
$$100p - 1000 = 1100 - 50p$$
$$p^e = 14$$

We can then plug this into either supply or demand to recover equilibrium quantity:

$$q^e = q_{tot}^{SR}(p^e)$$
$$q^e = 400$$

(c) Graph the market equilibrium and compute total short-run producer surplus in this case.

SOLUTION:



From the graph, the PS will be the shaded triangle. We can compute this simply using the area of a triangle:

$$PS = \frac{1}{2}400(14 - 10)$$
$$= 800$$

(d) Show that the total producer surplus you calculated in part (c) is equal to total industry profits plus industry short-run fixed costs.

SOLUTION: If the market supplies 400 units, one of the 100 identical firms will supply (400/100=4) units. We can plug this into each firm's profit function to get firm profit:

$$\begin{aligned}\pi(q) &= pq - TC(q)\\ &= 14 * 4 - (0.5(4)^2 + 10 * 4 + 5)\\ &= 3\end{aligned}$$

If each firm gets profit of 3, then industry profit is 3x100=300. Industry fixed cost is each firms FC, 5, times 100 firms, to get 500. We can confirm then that 300+500=800, as expected.

Problem 6

Suppose the government imposed a \$3 tax on snuffbox makers in the industry described in Problem 6.

(a) How would this tax change the market equilibrium? Please calculate the new supply function, demand function, equilibrium price, and equilibrium quantity. Show the change in a graph.

SOLUTION: After the imposition of a per unit tax, because it is assessed on firms, the firms will face a new total cost:

$$\begin{aligned}STC(q) &= 0.5q^2 + 10q + 5 + 3q\\ &= 0.5q^2 + 13q + 5\end{aligned}$$

Repeating the analysis from the previous question will yield a new firm-level supply:

$$q_i^{SR} = \begin{cases} p - 13 \; if \; p \geq 13 \\ 0 \; if \; p < 13 \end{cases}$$

And a new corresponding industry supply:

$$q_{tot}^{SR} = \begin{cases} 100p - 1300 \; if \; p \geq 10 \\ 0 \; if \; p < 10 \end{cases}$$

Demand is unchanged, so we find equilibrium by setting equal our new industry supply with the old demand:

$$\begin{aligned} q_{tot}^{SR} &= Q \\ 100p - 1300 &= 1100 - 50p \\ \Rightarrow p^e &= 16 \\ \Rightarrow q^e &= 300 \end{aligned}$$

(b) How would the burden of this tax be shared between snuffbox buyers and sellers?

SOLUTION: Before the tax, consumers paid \$14 for a snuffbox, and now they pay \$16, so their price went up \$2. Before the tax, firms received \$14 for a snuffbox, and they now receive \$16-\$3=\$13, so their price went down by \$1. Thus, the consumer share is \$2 and the producer share is \$1. Notice that although the tax is imposed on firms, the cost is split between both groups, and the share is actually larger for consumers.

(c) Calculate the total loss of producer surplus as a result of the taxation of snuffboxes. Show that this loss equals to the change in total short-run profits in the snuffbox industry. Why don't fixed costs enter into this computation of the change in short-run producer surplus?

SOLUTION: We again use a geometric argument to find the new PS:

$$\begin{aligned} PS' &= \frac{1}{2}300(13 - 10) \\ &= 450 \end{aligned}$$

Before the tax, PS was 800, so this represents a decrease of 350. We can again calculate profits by noticing that industry supply of 300 indicates that each firm supplies 3 units, and plugging this into our profit function:

$$\begin{aligned} \pi(q') &= pq' - TC(q') \\ &= 13 * 3 - (0.5(3)^2 + 10 * 3 + 5) \\ &= -0.5 \end{aligned}$$

The total industry profit will then be 100x-0.5=-50. Given that our old profits were 300, this is a loss in profits of 350, just the same as the decrease in PS. The fixed cost does not factor in at all because the FC does not change with the tax, and the FC does not factor into the short-run decision making of the firms either before or after the tax.