Problem Set 3 Solutions

Question 1.

a) When Robinson sets up the firm, we assume that he acts like any price taking firm in the market and that given wages and price, he will act to maximize profit. The firm's profit maximization problem is

$$\max_{l} \ \pi(p,l) = pf(l) - wl = p\beta\sqrt{l} - wl$$

Setting π_l equal to zero, we obtain the first order condition, which we solve for l to find $l^*(p,w)$.

$$\begin{aligned}
\pi_l = \frac{1}{2}p\beta l^{-1/2} - w &= 0 \\
\frac{1}{2}p\beta l^{-1/2} &= w \\
l^{-1/2} &= \frac{2w}{p\beta} \\
l &= \left(\frac{2w}{p\beta}\right)^{-2} \\
&= \left(\frac{p\beta}{2w}\right)^{2} \\
l^*(p,w) &= \left(\frac{p\beta}{2w}\right)^{2}
\end{aligned}$$

Plugging $l^*(p,w)$ back into $\pi(p,l)$, we obtain $\pi^*(p,w)$.

$$\begin{aligned}
\pi(p,l^*(p,w)) &= p\beta\sqrt{\left(\frac{p\beta}{2w}\right)^{2}} - w\left(\frac{p\beta}{2w}\right)^{2} \\
&= \frac{(p\beta)^2}{2w} - \frac{(p\beta)^2}{4w} \\
&= \frac{(p\beta)^2}{4w} \\
&= \pi^*(p,w)
\end{aligned}$$

b) The time constraint is,

$$R + L = T$$

and his budget constraint by

$$pC = wL + \pi$$

We can solve for labor in terms of T and R and substitute it into the budget.

$$\begin{aligned} L &= T - R \\ pC &= w(T - R) + \pi \\ pC + wR &= wT + \pi \\ pC + wR &= S \qquad \text{where } S \text{ is the full income, } wT + \pi \end{aligned}$$

c) Using the budget constrainst from Part (b), we can write Robinson's utlity maximization problem as

$$\max_{C,R}\{U(C, R) = \alpha \log(C) + (1 - \alpha) \log(R)\} \text{ such that } pC + wR = S$$

which gives the Lagrangian form,

$$\mathcal{L} = \alpha \log(C) + (1 - \alpha) \log(R) + \lambda[S - pC - wR]$$

the following first order conditions are then,

At optimum,

$$\begin{aligned} [C] &: \frac{\alpha}{C} = \lambda p \\ [R] &: \frac{1 - \alpha}{R} = \lambda w \\ [\lambda] &: pC + wR = S \end{aligned}$$

We can then solve the first order equations as a system of equations to obtain our C^*, R^*, and L^*.

$$
\begin{aligned}
\frac{\alpha}{C} &= \lambda p \\
\lambda &= \frac{\alpha}{pC} \\
\frac{1-\alpha}{R} &= \lambda w \\
\frac{1-\alpha}{R} &= \frac{\alpha}{pC} w \\
\frac{1-\alpha}{\alpha} &= \frac{R}{pC} w \\
pC + wR &= S \\
pC &= S - wR \\
\frac{1-\alpha}{\alpha} &= \frac{R}{S - wR} w \\
\frac{\alpha}{1-\alpha} &= \frac{S - wR}{wR} \\
\frac{\alpha}{1-\alpha} &= \frac{S}{wR} - 1 \\
\frac{S}{wR} &= 1 + \frac{\alpha}{1-\alpha} \\
\frac{S}{wR} &= \frac{1-\alpha}{1-\alpha} + \frac{\alpha}{1-\alpha} \\
\frac{S}{wR} &= \frac{1}{1-\alpha} \\
S(1-\alpha) &= wR
\end{aligned}
$$

$$
R^*(p, w, S) = \frac{S(1-\alpha)}{w}
$$

$$
\begin{aligned}
pC + w\left(\frac{S - \alpha S}{w}\right) &= S \\
pC + S - \alpha S &= S \\
pC &= \alpha S
\end{aligned}
$$

$$
C^*(p, w, S) = \frac{\alpha S}{p}
$$

$$
\begin{aligned}
L*(p, w, S) &= T - R^*(p, w, S) \\
&= T - \frac{S(1-\alpha)}{w}
\end{aligned}
$$

d) If we let $p = 1$ (we can think of this as a change of units that will not effect behavior), we can calculate the equilibrium price of labor w^*. First we can use that the amount demanded by Robinson in the labor market (C^*) must be equal to the amount supplied by the firm f. From our previous answers, we know,

$$\begin{aligned}
C^*(p^*, w^*, S^*) &= f(l^*(p^*, w^*)) \\
C^*(1, w^*, S^*) &= f(l^*(1, w^*)) \\
\alpha S &= f\left(\frac{\beta}{2w^*}\right)^2 \\
\alpha S &= \frac{\beta^2}{2w^*} \\
S &= \frac{\beta^2}{2\alpha w^*} \\
w^*T + \pi^*(1, w^*) &= \frac{\beta^2}{2\alpha w^*} \\
w^*T + \frac{\beta^2}{4w^*} &= \frac{\beta^2}{2\alpha w^*} \\
(w^*)^2 T + \frac{\beta^2}{4} &= \frac{\beta^2}{2\alpha} \\
(w^*)^2 T &= (\frac{1}{2\alpha} - \frac{1}{4})\beta^2 \\
(w^*)^2 T &= (\frac{2-\alpha}{4\alpha})\beta^2 \\
(w^*)^2 &= (\frac{2-\alpha}{4\alpha T})\beta^2 \\
w^* &= \beta\sqrt{\frac{2-\alpha}{4\alpha T}}
\end{aligned}$$

Now that we know wages, w^* it's simple to solve for optimal amounts,

$$
\begin{aligned}
L^* = l^*(1, w^*) &= \left(\frac{\beta}{2\beta\sqrt{\frac{2-\alpha}{4\alpha T}}} \right)^2 \\
&= \frac{1}{4\left(\frac{2-\alpha}{4\alpha T}\right)} \\
&= \frac{1}{\left(\frac{2-\alpha}{\alpha T}\right)} \\
&= \frac{\alpha T}{2-\alpha}
\end{aligned}
$$

$$
\begin{aligned}
R^* &= T - L^* \\
&= T - \frac{\alpha T}{2-\alpha} \\
&= \left(1 - \frac{\alpha}{2-\alpha}\right) T
\end{aligned}
$$

$$
\begin{aligned}
C^*(1, w^*, S^*) &= f(l^*) \\
&= \beta\sqrt{\frac{\alpha T}{2-\alpha}}
\end{aligned}
$$

e) We have what we need to take the partial derivatives,

$$
\begin{aligned}
w^*_\beta &= \sqrt{\frac{2-\alpha}{4\alpha T}} \\
w^*_\alpha &= -\frac{\beta}{2\alpha^2 T}\left(\frac{2-\alpha}{\alpha T}\right)^{-1/2} \\
w^*_T &= -\frac{\beta}{4T}\left(\frac{2-\alpha}{\alpha T}\right)^{-1/2}
\end{aligned}
$$

For w^*_β, we know that productivity in the economy increases, this should raise our wages (which are in terms of consumption) because the marginal product of labor is increasing.

For α, Robinson now cares less about leisure and more about consumption. This means that he will work more at any price, and so labor supply increases. An increase in supply drives down wages in the market.

For T, Robinson now has more time, so he will be willing to work more. Like with α, labor supply subsequently increases and wages in the market decrease.

f)

$$
\begin{aligned}
C^*(1, w^*, S^*) &= \beta\sqrt{\frac{\alpha T}{\alpha - 2}} \\
&= \beta\sqrt{T}\left(\frac{2}{\alpha} - 1\right)^{-1/2} \\
C^*_\alpha &= \beta\sqrt{T}\frac{1}{2}\left(\frac{2}{\alpha} - 1\right)^{-3/2} 2\log(\alpha) \\
C^*_\beta &= \sqrt{\frac{\alpha T}{2 - \alpha}} \\
C^*_T &= \beta\frac{1}{2}\left(\frac{\alpha T}{2 - \alpha}\right)^{-1/2}\left(\frac{\alpha}{2 - \alpha}\right)
\end{aligned}
$$

For C^*_α, Robinson cares more about consumption and will work more. This lowers wages (as we saw in part e)), but will also lower prices. In equilibrium, when he is working more, there must be more produced in the market, and thus more consumed.

For C^*_β, we know that increased productivity must increase supply and thus increase the quantity consumed in the market.

For C^*_T, it is again similar to α, labor supply has increase and this will lead to more produced and consumed in the market.

g)

$$
\begin{aligned}
R^* &= T - \frac{\alpha T}{2 - \alpha} \\
&= \left(1 - \frac{\alpha}{2 - \alpha}\right)T \\
&= T - \left(\frac{2}{\alpha} - 1\right)^{-1/2} \\
R^*_\alpha &= -\frac{1}{2}\left(\frac{1}{\alpha} - \frac{1}{2}\right)^{-3/2} 2\log(\alpha) \\
R^*_\beta &= 0 \\
R^*_T &= 1 - \frac{\alpha}{2 - \alpha}
\end{aligned}
$$

For R^*_α, Robinson cares more about consumption and will work more. This increases labor supply and means he will work more.

For R^*_β, increased production will make work more appealing because the wage is higher; however, leisure is also more appealing because income has increased. The income and substitution effect cancel and leisure remains constant.

For R^*_T, more time will mean more leisure.

h) Robinson decides that he will pick C^*, R^*, and L^* to maximize his utility rather acting as a price taker in the labor market and at the firm.

His new optimization problem (the social planner's problem) is

$$\max_{C,R,L}\{U(C,R) = \alpha\log(C) + (1-\alpha)\log(R)\} \text{ such that } C = f(L) = \beta\sqrt{L} \text{ and } L + R = T$$

which has the following Lagrangian.

$$\mathcal{L} = \alpha\log(C) + (1-\alpha)\log(R) + \lambda_C[\beta\sqrt{L} - C] + \lambda_T[T - L - R]$$

By taking partial derivatives of the Lagrangian in terms of C, R, and L, we obtain three first order conditions.

$$\begin{aligned}
&[C] \quad U_c = \frac{\alpha}{C} = \lambda_C \\
&[R] \quad U_R = \frac{1-\alpha}{R} = \lambda_T \\
&[L] \quad \lambda_C f' = \lambda_C \frac{\beta}{2\sqrt{L}} = \lambda_T \\
&[\lambda_C] \quad f(L) = \beta\sqrt{L} = C \\
&[\lambda_T] \quad T = L + R
\end{aligned}$$

We can, then, solve the first order conditions above as a system of equations to find our optimum.

$$
\begin{aligned}
\lambda_C \frac{\beta}{2\sqrt{L}} &= \lambda_T \\
\frac{\beta}{2\sqrt{L}} &= \frac{\lambda_T}{\lambda_C} \\
\frac{\beta}{2\sqrt{L}} &= \frac{\left(\frac{1-\alpha}{R}\right)}{\left(\frac{\alpha}{C}\right)} \\
\frac{\beta}{2\sqrt{L}} &= \frac{C(1-\alpha)}{\alpha R} \\
\alpha\beta R &= 2C\sqrt{L}(1-\alpha) \\
\alpha\beta R &= 2(\beta\sqrt{L})\sqrt{L}(1-\alpha) \\
\alpha\beta R &= 2\beta L(1-\alpha) \\
\alpha\beta R &= 2\beta L - 2\alpha\beta L \\
\alpha R &= 2L - 2\alpha L \\
\alpha R &= (2-2\alpha)L \\
\alpha(T-L) &= (2-2\alpha)L \\
\alpha T - \alpha L &= (2-2\alpha)L \\
\alpha T &= (2-\alpha)L
\end{aligned}
$$

$$
L^* = \frac{\alpha T}{2-\alpha}
$$

$$
\begin{aligned}
R^* &= T - L^* \\
&= T - \frac{\alpha T}{2-\alpha}
\end{aligned}
$$

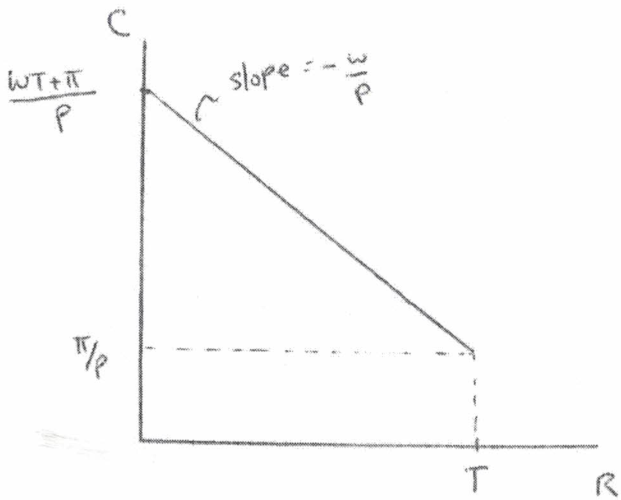
$$
\begin{aligned}
C^* &= f(L^*) \\
&= \beta\sqrt{\frac{\alpha T}{2-\alpha}}
\end{aligned}
$$

These results are the same as those from Question 1, Part (d). In other words, the market from Question 1 maximizes Robinson's utility, just as if he were to pick C^*, R^*, and L^* optimally himself.

Problem 2

This question concerns the Robinson Crusoe Economy where Robinson's utility function is given by $U(C,R) = C^{\alpha}R^{(1-\alpha)}$ for the consumption of good C and leisure R. He is endowed with T units of time. that he can allocate either to leisure (R) or to labor (L). Let $\boldsymbol{p}$ be the price of the good and $\boldsymbol{w}$ the per unit price of labor. Robinson also owns a firm that has access to a technology described by $f(l) = Al$ where $\boldsymbol{A}$ is a constant.

a. Draw the constraint of the optimization problem faced by Robinson on a graph with C on the y-axis and R on the x-axis. Make sure you label all relevant intercept/s and slope.



$$L + R = T \quad \& \quad pC = wL + \pi$$

$$\Rightarrow pC = w(T-R) + \pi$$

$$\Rightarrow C = \frac{wT+\pi}{p} - \frac{w}{p}R$$

Derive Robinson's optimal demands for C*(p,w,S) and R*(p,w,S) (and thus L*) where is full income as defined in class. You can assume that SOCS are satisfied.

b.

The lagrangian is given by: $\mathcal{L} = C^{\alpha}R^{(1-\alpha)} + \lambda\left[wT+\pi - pC - wR\right]$

FOC

$[C]: \alpha C^{\alpha-1}R^{1-\alpha} - \lambda p = 0 \Rightarrow \alpha C^{\alpha-1}R^{1-\alpha} = \lambda p$

$[R]: (1-\alpha)C^{\alpha}R^{-\alpha} - \lambda w = 0 \Rightarrow (1-\alpha)C^{\alpha}R^{-\alpha} = \lambda w$

$[\lambda]: wT + \pi - pC - wR = 0 \Rightarrow wT + \pi = pC + wR$

$[R]/[C]: \quad \frac{w}{p} = \frac{(1-\alpha)}{\alpha}\frac{C}{R} \Rightarrow \boxed{C = \frac{\alpha}{1-\alpha}\frac{w}{p}R} \quad (*)$

Substitute (*) into $[\lambda]$:

$$wT + \pi = p\left[\frac{\alpha}{1-\alpha}\frac{w}{p}R\right] + wR$$

$$\Rightarrow wT + \pi = w\left(\frac{\alpha R + R - \alpha R}{1-\alpha}\right)$$

$$\Rightarrow wT + \pi = \frac{wR}{1-\alpha} \Rightarrow \boxed{R^{*} = \frac{(1-\alpha)(wT+\pi)}{w}}$$

Substituting l^* back into ⊛:

$$C = \frac{\alpha}{(1-\alpha)}\frac{w}{p}\frac{(1-\alpha)(wT+\pi)}{w} \Rightarrow \boxed{C^* = \frac{\alpha(wT+\pi)}{p}}$$

we did not necessarily ask for L^* but it's easy to find:

$$L = T - \frac{(1-\alpha)(wT+\pi)}{w}$$

$$\Rightarrow L = \frac{wT - wT - \pi + \alpha wT + \alpha\pi}{w}$$

$$\Rightarrow \boxed{L^* = \alpha T - \frac{(1-\alpha)\pi}{w}}$$

c. Derive the firm's profit maximizing demand for labor for any prices.

we know that with linear production functions the usual mathematical approach does not help.

$\max_{l}\ pAl - wl \Rightarrow$ FOC: $[l]$: $pA - w = 0$ which cannot be solved for l.

Let's consider 3 different possibilities for p:

- $p < \frac{w}{A} \Rightarrow$ firm makes negative profit for every unit $\Rightarrow l^* = 0$
- $p > \frac{w}{A} \Rightarrow$ firm makes positive profit for every unit $\Rightarrow l^* = \infty$
- $p = \frac{w}{A} \Rightarrow$ firm make zero profit $\Rightarrow l^* \in [0, \infty)$

d. Solve for the equilibrium prices and quantities of this economy.

given the answer in part c), we cannot solve for the market clearing conditions as we usually do. But solving for the equilibrium is more intuitive and less mathematical.

- When $p < \frac{w}{A}$, this cannot be an equilibrium because firm's demand for labor is $l^* = 0$ but Robinson's supply is $L^* > 0$. Labor market does not clear
- When $p > \frac{w}{A}$, firm's labor demand is $l^* = \infty$ which obviously cannot clear the labor market.
- These leave us with $p = \frac{w}{A}$ or $\frac{w^*}{p^*} = A$ as potential candidate

Then, for the firm (as expected):

$$\pi^* = p \cdot A \cdot l - wl$$
$$= \frac{w}{A} \cdot A \cdot l - wl$$
$$= 0$$

For the consumer:

$$c^* = \frac{\alpha (wT + 0)A}{w} = \alpha TA$$

$$L^* = \alpha T - \frac{(1-\alpha)}{w} = \alpha T$$

When $P = \frac{w}{A}$, $l^* \in [0, \infty)$. Thus, $l^* = \alpha T$ will clear the labor market.

When $l^* = \alpha T$, $f(\alpha T) = \alpha T A$ which also clears the consumption good market.

e. Thanks to technological progress the new production function is given by $f(l) = Bl$ where $B > A$. Explain how the equilibrium in the economy will change.

First note that $L^* = \alpha T$ is unchanged while $c^* \uparrow$ to $C^* = \alpha T B$ because $B > A$.

But even though labor is unchanged at $L^* = l^* = \alpha T$ higher consumption can be supported because of technological progress:

$$f(\alpha T) = \alpha T B.$$

Finally we just need prices to adjust to $\frac{w^*}{p^*} = B > A$.

Question 3

(a) The consumer maximizes

$$\begin{aligned} \max_{C,R,L} \quad & C^\alpha R^\beta \\ \text{s.t.} \quad & C = wL + r + \tfrac{\pi}{100} \\ & L + R = 1 \end{aligned}$$

The two constraints can be combined into

$$C + wR = w + r + \frac{\pi}{100}$$

where the right-hand side is the new version of the full income seen in class. We can thus write the Lagrangian as

$$\mathcal{L} = C^\alpha R^\beta + \lambda \left[w + r + \frac{\pi}{100} - C - wR \right]$$

FOC's:

$$\begin{aligned} [C]: \quad & \alpha C^{\alpha-1} R^\beta = \lambda \\ [R]: \quad & \beta C^\alpha R^{\beta-1} = \lambda w \\ [\lambda]: & \; C + wR = w + r + \frac{\pi}{100} \end{aligned}$$

$[R]/[C]$ implies

$$\frac{\beta}{\alpha}\frac{C}{R} = w \implies C = \frac{\alpha}{\beta} R w.$$

Plugging into $[\lambda]$, we get

$$\frac{\alpha}{\beta} R w + wR = w + r + \frac{\pi}{100}$$

$$R^*(w, r, \pi) = \left(\frac{\beta}{\alpha + \beta} \right) \frac{1}{w} \left[w + r + \frac{\pi}{100} \right]$$

and therefore

$$C^*(w, r, \pi) = \left(\frac{\alpha}{\alpha + \beta} \right) \left[w + r + \frac{\pi}{100} \right]$$

$$L^*(w, r, \pi) = 1 - \left(\frac{\beta}{\alpha + \beta} \right) \frac{1}{w} \left[w + r + \frac{\pi}{100} \right]$$

(b) The firm wants to maximize

$$\max_{k,\ell} k^{\frac{1}{2}} \ell^{\frac{1}{2}} - rk - w\ell$$

FOC's

$$\begin{aligned} [k]: \quad & \frac{1}{2} k^{-\frac{1}{2}} \ell^{\frac{1}{2}} = r \\ [\ell]: \quad & \frac{1}{2} k^{\frac{1}{2}} \ell^{-\frac{1}{2}} = w \end{aligned}$$

(c) Plugging the FOC's into the profit function, we get

$$\pi = k^{\frac{1}{2}} \ell^{\frac{1}{2}} - \frac{1}{2} k^{-\frac{1}{2}} \ell^{\frac{1}{2}} k - \frac{1}{2} k^{\frac{1}{2}} \ell^{-\frac{1}{2}} \ell = k^{\frac{1}{2}} \ell^{\frac{1}{2}} - \frac{1}{2} k^{\frac{1}{2}} \ell^{\frac{1}{2}} - \frac{1}{2} k^{\frac{1}{2}} \ell^{\frac{1}{2}} = 0$$

From $[k]/[\ell]$, we get that

$$\frac{k}{\ell} = \frac{w}{r}.$$

The market clearing condition for labor is $\ell^* = 100L*$ and for capital is $k^* = 100K^*$. Using those two conditions, we find

$$\frac{w}{r} = \frac{100}{100L^*} \implies L^* = \frac{r}{w}.$$

Using the expression for L^* from part (a), we get

$$\frac{r}{w} = 1 - \left(\frac{\beta}{\alpha + \beta}\right)\frac{1}{w}\left[w + r\right],$$

where I also used the fact that $\pi = 0$. Solving for w, we get (after some algebra)

$$w = \frac{\alpha + 2\beta}{\alpha} r.$$

The consumption market clearing condition is

$$100C^* = (\ell^* k^*)^{\frac{1}{2}}.$$

Again using the expressions from (a), we get

$$100\left(\frac{\alpha}{\alpha + \beta}\right)\left[w + r\right] = \left(100 \times 100\frac{r}{w}\right)^{\frac{1}{2}}$$

Plugging in $w = \frac{\alpha+2\beta}{\alpha} r$, we can solve for r^* as

$$r^* = \frac{1}{2}\left(\frac{\alpha}{\alpha + 2\beta}\right)^{\frac{1}{2}}$$

and

$$w^* = \frac{1}{2}\left(\frac{\alpha + 2\beta}{\alpha}\right)^{\frac{1}{2}}.$$

Finally,

$$L^* = \frac{r^*}{w^*} = \frac{\alpha}{\alpha + 2\beta}$$

and

$$\ell^* = 100L^* = 100\frac{\alpha}{\alpha + 2\beta}.$$

The question also asks for K^*, but it should be $K^* = 1$, since K does not enter into the consumer's utility function. We can also pretty easily find

$$R^* = 1 - L^* = 1 - \frac{\alpha}{\alpha + 2\beta} = \frac{2\beta}{\alpha + 2\beta}.$$

Problem 4

a

The monopolist's profit maximization problem is:

$$\max_y \{(100 - y)y - 5y\}$$

The FOC is:

$$[y]: \quad 100 - 2y - 5 = 0 \Rightarrow y^* = \frac{95}{2}, \quad p^* = \frac{105}{2}$$

so that profits are

$$\pi^* = \frac{95}{2}\frac{105}{2} - 5\frac{95}{2} = \frac{9975 - 950}{4} = \frac{9025}{4}$$

The second-order condition gives us $-2 < 0$, so we know that profits are in fact maximized for this output choice.

b

The profit maximization problem is now:

$$\max_y \left\{(100 - y)y - 300 + 5y - \frac{1}{4}y^2\right\}$$

which gives FOC

$$[y]: \quad 100 - 2y + 5 - \frac{1}{2}y = 0 \rightarrow 105 = \frac{5}{2}y \Rightarrow y^* = 42, \quad p^* = 58$$

This gives profits of

$$\pi^* = 58 \times 42 - \left(300 - 5 \times 42 + \frac{1}{4}(42)^2\right) = 1905$$

c

See Figure 1.

d

No, as we saw in class the quantity produced by the monopolist is not efficient. In particular for units between the monopoly quantity and where the demand curve intersects the monopolist's marginal cost curve, the

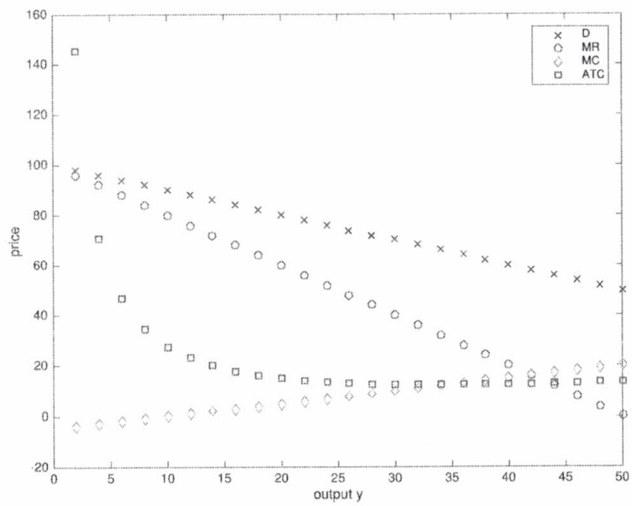


Figure 1: Demand, marginal revenue, and cost curves for problem 1. Monopoly profits are equal to the area of the rectangle with base equal to the number of units sold, and with height equal to the difference between the height of the demand curve and the height of the ATC curve at the monopoly quantity.

consumer's willingness to pay (or marginal value) is above the cost for the units to be produced, but the units are not being sold.

e

The price elasticity of demand is

$$|\varepsilon_d| = |\frac{\frac{dy}{y}}{\frac{dp}{p}}| = |\frac{dy}{dp}\frac{p}{y}| = |(\text{-}1\ )\frac{58}{42}| \approx 1.4 > 1$$

so that the monopolist is operating in the elastic region of the demand curve.

Problem 5

a

The profit maximization problem is given by

$$\max_{y} \; p(y)y - C(y) - 1000$$

Using the inverse market demand function (that is, we need to invert the demand function given in the problem to express price as a function of quantity sold), we rewrite this as

$$\max_{y} \; (100 - y)y - 1000 = \max_{y} \; 100y - y^2 - 1000$$

b

The FOC is

$$[y]: \; 100 - 2y = 0 \Rightarrow y^* = 50$$

Checking the SOC, we get $-2 < 0$, so we have a maximum. Note that we have the usual interpretation of $MR = MC$ here. In this case $MR = 100 - 2y$ and $MC = 0$. The price is $p(y) = 100 - y = 100 - 50 = 50 = p^*$. Profit is: $\pi = 50 \times 50 - 1000 = 1500$. See attached figure.

c

No, the solution in part b is not efficient. In particular, for units between $y^* = 50$ and $y = 100$, WTP by consumers is higher than the MC of 0, but the units are not sold. We calculate: (See attached figure)

$$PS = 50 * 50 = 2500$$

$$CS = \frac{50 * 50}{2} = 1250$$

$$DWL = \frac{50 * 50}{2} = 1250$$

Note that PS does not include fixed costs, so in this case $PS \neq \pi$.

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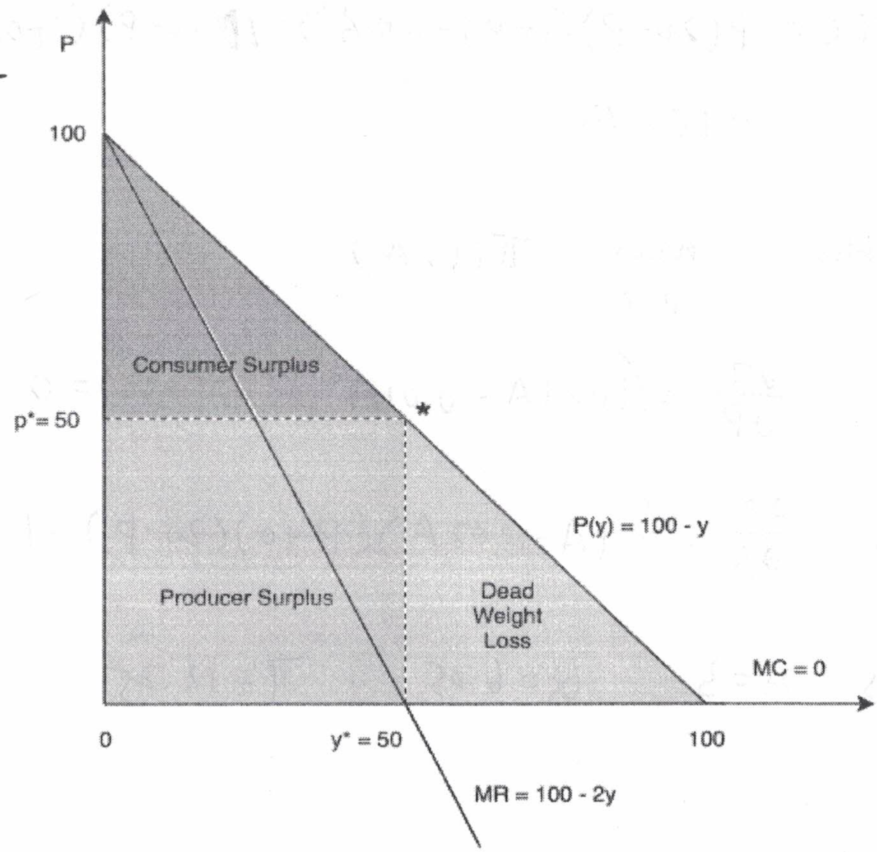
d

$$\varepsilon_D = \frac{\frac{dy}{y}}{\frac{dp}{p}} = \frac{dy}{dp}\frac{y}{p} = (-1) * \frac{50}{50} = -1 * 1 \Rightarrow |\varepsilon_D| = 1, \text{ from } y(p) = 100 - p$$

In other words, the monopolist is operating on the portion of the demand curve that is unit elastic - in between the elastic and inelastic parts of the demand curve. Therefore we can confirm that the monopolist is not operating on the inelastic portion of the demand curve.

Problem 5



Problem 6

(a) When $A=0$ $\quad Q = 20-P \Rightarrow P = 20-Q$

$TR(Q) = Q(20-Q) \Rightarrow \quad MR(Q) = 20-2Q.$

$TC(Q) = 10Q+15 \Rightarrow \quad MC(Q) = 10$

To maximize profit, $\quad MR(Q) = MC(Q) \Rightarrow 20-2Q = 10 \Rightarrow Q=5$

Thus $P=10$ $\quad \pi = 15\times 5 - (10\times 5+15) = 10$

(b) $TR(P) = P(20-P)(1+0.1A-0.01A^2)$

$TC(P) = 10(20-P)(1+0.1A-0.01A^2)+15+A$

$$\pi = TR - TC = P(20-P)(1+0.1-0.01A^2) - 10(20-P)(1+0.1A-0.01A^2)$$
$$-15-A$$

Firm's problem $\max_{P,A} \pi(P, A)$

FOC: [P] $\frac{\partial \pi}{\partial P} = (1+0.1A-0.01A^2)(-2P+30)=0$

[A] $\frac{\partial \pi}{\partial A} = (0.1A-0.02A)(P-10)(20-P)-1=0$

$\Rightarrow$. $P=15$ $A=3$ $Q=6.05$ $\pi=12.25$

Problem 7

(a) $MC=12$ $TR=PQ=(20-\frac{1}{50}Q)Q \Rightarrow MR=20-0.04Q$

In monopoly $MR=MC \Rightarrow 20-0.04Q=12 \Rightarrow Q^M=200$ $P^M=16$

In perfect competition $P=MC \Rightarrow 20-0.02Q=10$

$\Rightarrow Q^C=500$ $P^C=10$

(b) Consumer surplus in monopoly

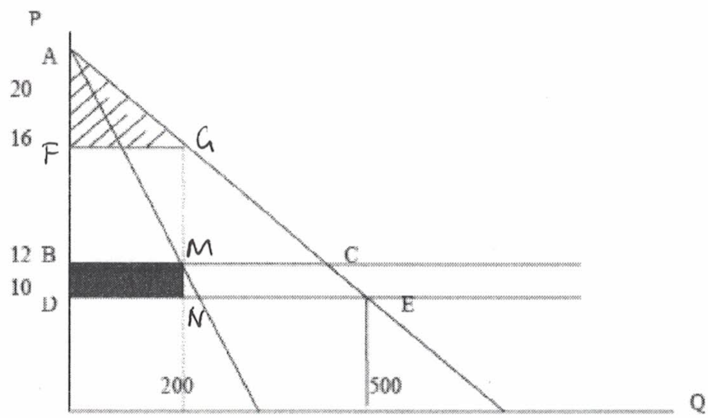
$CS^M = 0.5\times(20-16)\times 200 = 400$

Consumer surplus in perfect competition

$CS^C = 0.5\times(20-10)\times 500 = 2500$

The loss of consumer surplus from monopolization is 2100.

C.



AFG is consumer's surplus in monopoly

ADE is the consumer surplus in perfect competition

BDNM is the cost payed to lobbyists.

Here the marginal cost is higher in monopoly compared with that in perfect competition.

Thus monopoly decreases consumer surplus even more.